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FOUNDATIONS OF STATISTICAL CRACK MECHANICS

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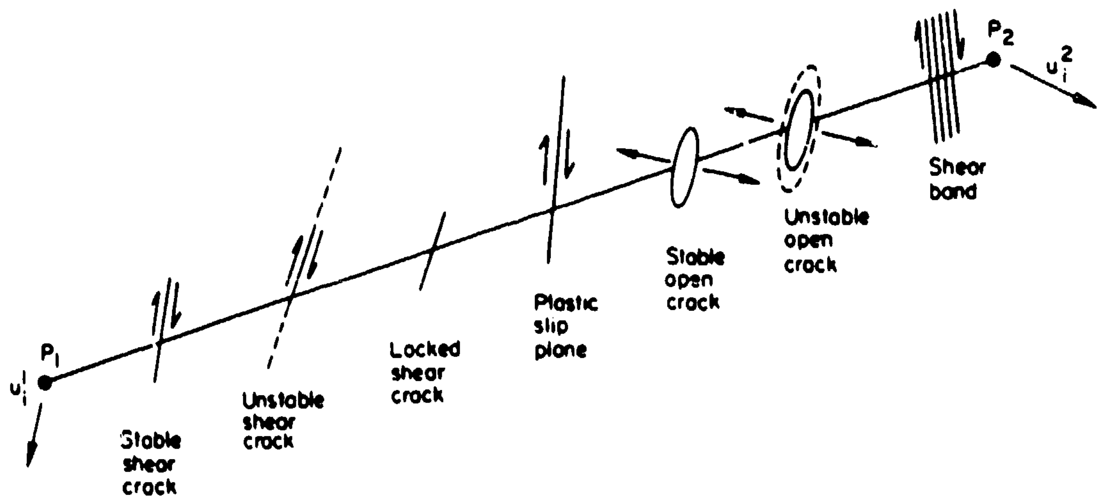
The principle of superposition of elastic and plastic strain rates in regions of plastic flow was hypothesized by Reuss [1] in order to provide a smooth transition of the stresses between regions of elastic and elastic-plastic deformation. In this paper a generalization of this hypothesis is derived which allows for the formulation of more complex constitutive laws that can account for simultaneous plastic flow, fracture, fragmentation, and changes in porosity. Such processes can occur during impact, explosions and earthquakes. The possibility of analyzing such processes by finite element methods has been made possible by the advent of large scale computers, but most practioners agree that more general constitutive laws are required to account adequately for the complex flow processes involved. In particular, many fracture processes in brittle materials are thought to depend on the strain rate and size of the samples involved. These effects are accounted for in the current approach without the introduction of special hypotheses. The method also accounts for many aspects of finite deformation, and has been used in problems involving very large distortions. The results given here are essentially the same as obtained by Dienes and Margolin [2], but in that earlier derivation it was necessary to make ad hoc hypotheses which are not necessary in the current approach.

Consider two adjacent points P_2 and P_1 in a complex material, illustrated in the Figure, which are separated by a variety of crystals and flaws. The difference in velocity of these points can be written

$$u_i^2 - u_i^1 = \sum \Delta x_j u_{i,j}^c + \sum \Delta u_i + \sum \dot{n} \Delta v_i \qquad (1)$$

with the first term on the right denoting the continuum contribution, the second denoting the effect of cracks, and the third denoting the the influence of dislocation motion. The first term can be treated



using the methods of continuum mechanics, but the second term needs to be transformed to make it computationally useful. The treatment of the third term belongs to the theory of dislocations and plasticity, and has been discussed elsewhere [3,4].

The sum over discrete flaws in the second term needs to be replaced by a sum (in the limit, an integral) over a statistical crack distribution, so that

$$\sum \Delta u_i = \sum \Delta s \Delta c \Delta\Omega (\bar{N} \delta u_i + \dot{\bar{N}} \delta v_i) \tag{2}$$

where $\bar{N}$ denotes the number of cracks per unit arc length with orientation Ω and size c, Δs denotes the element of arc in reference coordinates, Δc the range of crack sizes and $\Delta\Omega$ the range of crack orientations for the selected subdivision of $\bar{N}$, and δu_i and δv_i denote the discontinuity in velocity and displacement across a crack of size c and orientation Ω. Now, it is straightforward to show, as indicated by Dienes [5], that

$$\bar{N} = -a \cos \theta \, N_{c\Omega} \tag{3}$$

where a is the cross section of the crack, θ is the angle of its normal with the reference direction for $\bar{N}$, and N is the number of cracks with orientation Ω exceeding c in size. The negative sign appears because N decreases with increasing c. The element of solid angle is written

$$\Delta\Omega = \sin\theta \, d\theta \, d\phi \tag{4}$$

for compactness, with θ and ϕ the usual polar angles. The subscripts c and Ω on N denote differentiation. The only really useful choice for Δs is to take it as one of the reference directions ΔX_k in the reference coordinates of the undisturbed continuum [6]. Then, by dividing by Δx_j, taking the limit and using the standard notation $F = VR$ for polar decomposition

$$X_{k,j} = F^{-1}_{kj} = R_{sk} V^{-1}_{sj} \quad . \tag{5}$$

The crack normals in current and reference coordinates are related by

$$R_{sk}\bar{n}_k = n_s \ , \ \bar{n}_k = \cos\theta \tag{6}$$

where the bars denote the normal in reference (material) coordinates. From this we can obtain the expression

$$u^d_{i,j} = -V^{-1}_{sj} \sum \Delta c \Delta\Omega n_s \left[a\delta\dot{v}_i N_{c\Omega} + \left(\dot{a} N_{c\Omega} + a\dot{N}_{c\Omega} \right) \delta v_i \right] \tag{7}$$

for the stretching due to crack opening, shear, and growth. Note that the crack distribution is considered given in reference coordinates, in which it remains constant. The change in displacement across a penny-shaped crack of radius c can be represented as the sum

$$\delta v_i = \delta v^o_i + \delta v^s_i \tag{8}$$

of an opening displacement parallel to the crack normal δv^o_i, given by Sack [7], and a shearing displacement parallel to the crack plane, given by Segedin [8], δv^s_i. The former is given by

$$\delta v^o_i = \gamma^o \sigma_n n_i f \tag{9}$$

where σ_n is the normal component of traction, and

$$f = \left(1 - r^2/c^2 \right)^{\frac{1}{2}} \tag{10}$$

gives the variation of displacement with distance r from the crack center. The average value of f is 2/3. Thus it is convenient to define a new constant β^o by

$$\gamma^o a = \frac{8}{3}\,\frac{1-\nu}{\mu}\,c^3 = \beta^o c^3 \quad . \tag{11}$$

making the assumption that cracks are circular. The average shear displacement is given by

$$\delta v_i^s = \gamma^s (T_i' - \tau_i) \tag{12}$$

where T_i' denotes the tangential component of traction acting in the far field on a plane parallel to the crack,

$$T_i' = (\delta_{ij} - n_i n_j)\sigma_{jk} n_k \tag{13}$$

and τ_i denotes the interfacial traction. Analogously, it is convenient to define a shear constant β^s by

$$\gamma^s a = \frac{16}{3}\,\frac{1-\nu}{2-\nu}\,\frac{c^3}{\mu} = 2\beta^s c^3 \quad . \tag{14}$$

Now, the velocity gradient can be expressed as the sum of a symmetric part, the stretching d_{ij}, and an antisymmetric part, the vorticity w_{ij}, so that

$$u_{i,j} = d_{ij} + w_{ij} \quad . \tag{15}$$

Then the average contribution to the velocity gradient due to crack opening is

$$d_{ij}^o = -\beta^o V_{ij}^{-1} (\hat{\sigma}_{k\ell} H^o + \sigma_{k\ell} G^o) a_{ijk\ell} \tag{16}$$

where the stress rate $\hat{\sigma}_{k\ell}$ is given by Dienes [6],

$$H^o = \sum N_{c\Omega}^o c^3 \Delta c \Delta\Omega \quad , \tag{17}$$

$$G^o = \sum (\dot{N}_{c\Omega}^o c^3 + 2N_{c\Omega}^o c^2 \dot{c}) \Delta c \Delta\Omega \tag{18}$$

and

$$a_{ijk\ell} = n_i n_j n_k n_\ell \quad . \tag{19}$$

However, to get the stretching due to crack shearing requires that the symmetric part of the velocity gradient be computed, yielding the result

$$d^s_{ij} = -\beta^s V^{-1}_{ij}(\hat{\sigma}_{k\ell}H^s + \sigma_{k\ell}G^s)b_{ijk\ell} \tag{20}$$

where

$$H^s = \sum N^s_{c\Omega}c^3 \Delta c \Delta\Omega \quad , \tag{21}$$

$$G^s = \sum (\dot{N}^s_{c\Omega}c^3 + 2N^s_{c\Omega}c^2\dot{c})\Delta c\Delta\Omega \quad , \tag{22}$$

and

$$b_{ijk\ell} = \delta_{i\ell}n_j n_k + \delta_{j\ell}n_i n_k - 2n_i n_j n_k n_\ell \quad . \tag{23}$$

These equations can be put in the form

$$A\dot{\sigma} + B\sigma = C \tag{24}$$

where A, B, and C are matrices involving stress and strain rate. This can be solved for $\dot{\sigma}$ numerically in the course of finite element calculations. The computation of the statistical distribution function $N(c,\Omega)$ of crack sizes by means of a Liouville equation is discussed by Dienes [9]. The result of such a computation for anisotropic rock has been illustrated in an example involving explosions in oil shale [10].

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